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ON THE INVARIANCE EQUATION FOR HEINZ MEANS

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Abstract. We solve the so-called invariance equation in the class of Heinz means, that is, we give necessary and sufficient conditions for the constants $0 \leqslant p,q,r \leqslant 1$ such that the identity

$$H_p(H_q(x,y),H_r(x,y)) = H_p(x,y) \quad (x,y \in \mathbb{R}^+)$$

holds true where the Heinz mean H_p is defined for $0 \leqslant p \leqslant 1$ as

$$H_p(x,y) = \frac{x^p y^{1-p} + x^{1-p} y^p}{2}.$$

The Taylor expansion of the Heinz mean is used.

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