

CHARACTERIZATIONS OF OPERATOR ORDER FOR k STRICTLY POSITIVE OPERATORS

JIAN SHI AND ZONGSHENG GAO

Abstract. Let A_i ($i = 1, 2, \dots, k$) be bounded linear operators on a Hilbert space. This paper aims to show a characterization of operator order $A_k \geq A_{k-1} \geq \dots \geq A_2 \geq A_1 > 0$ in terms of operator inequalities. Afterwards, an application of the characterization is given to operator equalities due to Douglas's majorization and factorization theorem.

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