

## INEQUALITIES FOR WEIGHTED SUMS OF POWERS AND THEIR APPLICATIONS

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*Abstract.* Two inequalities for weighted sums of powers are established. Applications to Jacobian elliptic functions and Legendre's elliptic integrals of the first kind are presented. Some known and new inequalities for circular and hyperbolic functions are obtained. Applications to certain iterative means including Gauss' arithmetic-geometric mean and Schwab-Borchardt mean are included.

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