

SOME SHARP INEQUALITIES INVOLVING SEIFFERT AND OTHER MEANS AND THEIR CONCISE PROOFS

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Abstract. In the paper, by establishing the monotonicity of some functions involving the sine and cosine functions, the authors provide a unified and concise proof of some known inequalities and find some new sharp inequalities involving the Seiffert, contra-harmonic, centroidal, arithmetic, geometric, harmonic, and root-square means of two positive real numbers a and b with $a \neq b$.

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