

## ANOTHER APPROACH FOR SANO'S CHARACTERIZATION OF THE $J$ -CHAOTIC ORDER

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*Abstract.* A selfadjoint involutive matrix  $J$  endows  $\mathbb{C}^n$  with the indefinite Krein space structure endowed by the inner product  $[\cdot, \cdot]$  given by  $[x, y] := \langle Jx, y \rangle$ ,  $x, y \in \mathbb{C}^n$ . For a pair of  $J$ -selfadjoint matrices  $A, B$  with positive eigenvalues,  $\text{Log}A \geq^J \text{Log}B$  is called the  $J$ -chaotic order or the indefinite chaotic order. Sano [8], proved as an application of Furuta inequality of indefinite type that  $\text{Log}A \geq^J \text{Log}B$  if and only if  $A^r \geq^J (A^{\frac{r}{p}} B^p A^{\frac{r}{p}})^{\frac{r}{p+r}}$  for all  $p > 0$  and  $r > 0$ . In this paper, we prove Sano's result using a different approach. In the process, some other results due to Bebiano, Lemos, Providência and Soares [4, 9] are reobtained. The techniques in this paper are inspired by [5].

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