Mathematical
Inequalities
& Applications
Volume 16, Number 1 (2013), 1–19
doi:10.7153/mia-16-01

NECESSARY AND SUFFICIENT CONDITIONS FOR BOUNDEDNESS OF THE HARDY–TYPE OPERATOR FROM A WEIGHTED LEBESGUE SPACE TO A MORREY–TYPE SPACE

V. I. BURENKOV AND R. OINAROV

Abstract. Necessary and sufficient conditions on functions u and w are established ensuring boundedness of the multi-dimensional Hardy-type operator $H_{n,\varphi}$ from a weighted Lebesgue space $L_{p,u}(\mathbb{R}^n)$ to a local Morrey-type space $LM_{q\theta,w}(\mathbb{R}^n)$ for a wide range of the numerical parameters p, q, θ.

Mathematics subject classification (2010): 47B38.

Keywords and phrases: Hardy–type operator, boundedness, weighted Lebesque spaces, general Morrey–type spaces.

REFERENCES

[1] V. I. BURENKOV, *Sobolev Spaces on Domains*, B. G. Teubner, Stuttgart-Leipzig, 1998, 312 pp.

[2] V. I. BURENKOV, A. GOGATISHVILI, V. S. GULIYEV, R. MUSTAFAEV, *Boundedness of the fractional maximal operator in local Morrey-type spaces,* Complex Analysis and Elliptic Equations **55**, 8–10 (2010), 739–758.

[3] V. I. BURENKOV, H. V. GULIYEV, *Necessary and sufficient conditions for boundedness of the maximal operator in the local Morrey-type spaces*, Studia Math. **163**, 2 (2004), 157–176.

[4] V. I. BURENKOV, H. V. GULIYEV, V. S. GULIYEV, *Necessary and sufficient conditions for boundedness of the fractional maximal operator in the local Morrey-type spaces*, J. Comput. Appl. Math. **208** (2007), 280–301.

[5] V. I. BURENKOV, V. S. GULIYEV, *Necessary and sufficient conditions for boundedness of the Riesz potential in local Morrey-type spaces,* Potential Anal. **30**, 3 (2009), 211–249.

[6] V. I. BURENKOV, V. S. GULIYEV, A. SERBETCHI, T. V. TARARYKOVA, *Necessary and sufficient conditions for boundedness of the genuine singular integral operators in the local Morrey-type spaces,* Eurasian Mathematical Journal **1**, 1 (2010), 32–53.

[7] V. I. BURENKOV, P. JAIN, T. V. TARARYKOVA, *On boundedness of the Hardy operator in Morrey-type spaces,* Eurasian Mathematical Journal **2**, 1 (2011), 52–80.

[8] D. E. EDMUNS, W. D. EVANS, *Hardy Operators, Function Spaces and Embeddings,* Springer, Berlin. 2004, 326pp.

[9] I. GENEBASHVILI, A. GOGATISHVILI, V. KOKILASHVILI, M. KRBEC, *Weight theory for integral transforms on spaces of homogeneous type*, Pitman Monographs and Surveys in Pure and Applied Mathematics 92, Longman, 1998.

[10] A. KUFNER, L. MALIGRANDA, L.-E. PERSSON, *The Hardy Inequality. About its History and Some Related Results,* Vydavatelsky Servis Publishing House, Pilzen, 2007, 162 pp.

[11] A. KUFNER, L.-E. PERSSON, *Weighted Inequalities of Hardy Type,* World Scientific, New Jersey-London-Singapore-Hong Kong, 2003, xviii+357 pp.

[12] C. B. MORREY, *On the solution of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc. **43** (1938), 126–166.

[13] R. OINAROV, *Two-sided norm estimates for certain classes of integral operators*, Trudy Mat. Inst. Steklov. **204** (1993), 240–250; English transl. in Proc. Steklov Inst. Math. **204**, 3 (1994), 205–214.

[14] L.-E. PERSSON, N. SAMKO, *The Weighted Hardy and potential operators in the generalized Morrey spaces*, J. Math. Anal. Appl. **350** (2009), 56–72.

Paper MIA-16-01

[15] N. SAMKO, *The Weighted Hardy and singular operators in Morrey spaces*, J. Math. Anal. Appl. **377** (2011), 792–806.
[16] G. SINNAMON, V. D. STEPANOV, *The Weighted Hardy Inequality: new proofs and the case* $p = 1$, J. London Math. Soc. **54**, 2 (1996), 89–101.