ON FRACTIONAL SMOOTHNESS OF
FUNCTIONS RELATED TO p–VARIATION

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Abstract. This paper is concerned with the study of two functionals of variational type - the Riesz type generalized variation $v_{p,\alpha}(f)$ $(1 < p < \infty, 0 \leqslant \alpha \leqslant 1 - 1/p)$ and the moduli of p-continuity $\omega_{1-1/p}(f;\delta)$. These functionals generate scales of spaces connecting the class V_p of functions of bounded p-variation and the Sobolev space W_p^1. Some limiting relations in these scales are proved. Sharp estimates of $v_{p,\alpha}(f)$ in terms of $\omega_{1-1/p}(f;\delta)$ are obtained.

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