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L^p CHANGE OF VARIABLES INEQUALITIES ON MANIFOLDS

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Abstract. We prove two-sided inequalities for the L^p-norm of a pushforward or pullback (with respect to an orientation-preserving diffeomorphism) on oriented volume and Riemannian manifolds. For a function or density on a volume manifold, these bounds depend only on the Jacobian determinant, which arises through the change of variables theorem. For an arbitrary differential form on a Riemannian manifold, however, these bounds are shown to depend on more general "spectral" properties of the diffeomorphism, using an appropriately defined notion of singular values. These spectral terms generalize the Jacobian determinant, which is recovered in the special cases of functions and densities (i.e., bottom and top forms).

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