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SHARP MAXIMAL INEQUALITIES FOR CONTINUOUS–PATH SEMIMARTINGALES

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Abstract. Let $\alpha \geqslant 0$ be a fixed number and let X, Y be continuous-path semimartingales such that Y is α-differentially subordinate to X and X is either a nonnegative supermartingale, or a nonnegative submartingale. We introduce a method which enables us to derive the best constants in the inequality between the first moments of Y and the maximal function of X. This generalizes the previous results of Burkholder and the author. As an application, we obtain sharp versions of some maximal estimates for stochastic integrals and Itô processes.

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