

ON LYAPUNOV-TYPE INEQUALITY FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract. In this paper, we will try to find a new Lyapunov-type inequality for a class of nonlinear systems, special cases of which contain some well-known Hamiltonian system, Emden-Fowler, half-linear and linear differential equations of second order. Our result extends the Lyapunov-type inequality given in [X. Wang, Stability criteria for linear periodic Hamiltonian systems, *J. Math. Anal. Appl.* 367 (2010), 329-336.].

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REFERENCES

- [1] R. C. BROWN, D. B. HINTON, *Opial's inequality and oscillation of 2nd order equations*, *Proc. Amer. Math. Soc.* **125** (1997), 1123–1129.
- [2] R. C. BROWN, D. B. HINTON, *Lyapunov inequalities and their applications*, in: T.M. Rassias (Ed.), *Survey on Classical Inequalities*, Kluwer, Dordrecht, 2000, pp. 1–25.
- [3] S. S. CHENG, *Lyapunov inequalities for differential and difference equations*, *Fasc. Math.* **23** (1991), 25–41.
- [4] D. ÇAKMAK, *Lyapunov-type integral inequalities for certain higher order differential equations*, *Appl. Math. Comput.* **216** (2010), 368–373.
- [5] D. ÇAKMAK, A. TIRYAKI, *Lyapunov-type inequality for a class of Dirichlet quasilinear systems involving the (p_1, p_2, \dots, p_n) -Laplacian*, *J. Math. Anal. Appl.* **369** (2010), 76–81.
- [6] D. ÇAKMAK, A. TIRYAKI, *On Lyapunov-type inequality for quasilinear systems*, *Appl. Math. Comput.* **216** (2010), 3584–3591.
- [7] O. DOŠLÝ, P. ŘEHÁK, *Half-linear differential equations*, *Mathematics Studies* 202, North-Holland 2005.
- [8] S. B. ELIASON, *A Lyapunov inequality*, *J. Math. Anal. Appl.* **32** (1970), 461–466.
- [9] G. SH. GUSEINOV, B. KAYMAKÇALAN, *Lyapunov inequalities for discrete linear Hamiltonian system*, *Comput. Math. Appl.* **45** (2003), 1399–1416.
- [10] G. SH. GUSEINOV, A. ZAFER, *Stability criteria for linear periodic impulsive Hamiltonian systems*, *J. Math. Anal. Appl.* **335** (2007), 1195–1206.
- [11] P. HARTMAN, *Ordinary differential equations*, Wiley, New York, 1964 and Birkhäuser, Boston 1982.
- [12] L. JIANG, Z. ZHOU, *Lyapunov inequality for linear Hamiltonian systems on time scales*, *J. Math. Anal. Appl.* **310** (2005), 579–593.
- [13] M. K. KWONG, *On Lyapunov's inequality for disfocality*, *J. Math. Anal. Appl.* **83** (1981), 486–494.
- [14] C. LEE, C. YEH, C. HONG, R. P. AGARWAL, *Lyapunov and Wirtinger inequalities*, *Appl. Math. Letters* **17** (2004), 847–853.
- [15] A. M. LIAPUNOV, *Probleme général de la stabilité du mouvement*, (French Translation of a Russian paper dated 1893) *Ann. Fac. Sci. Univ. Toulouse* **2** (1907), 27–247; Reprinted as *Ann. Math. Studies*, No. 17, Princeton, 1947.
- [16] P. L. NAPOLI, J. P. PINASCO, *Estimates for eigenvalues of quasilinear elliptic systems*, *J. Differential Equations* **227** (2006), 102–115.

- [17] B. G. PACHPATTE, *On Lyapunov-type inequalities for certain higher order differential equations*, J. Math. Anal. Appl. **195** (1995), 527–536.
- [18] B. G. PACHPATTE, *Lyapunov type integral inequalities for certain differential equations*, Georgian Math. J. **4**, 2 (1997), 139–148.
- [19] S. PANIGRAHI, *Lyapunov-type integral inequalities for certain higher order differential equations*, Electron. J. Differential Equations, Vol. **2009**, 28 (2009), 1–14.
- [20] N. PARHI, S. PANIGRAHI, *On Liapunov-type inequality for third-order differential equations*, J. Math. Anal. Appl. **233** (1999), 445–460.
- [21] A. TIRYAKI, M. ÜNAL, D. ÇAKMAK, *Lyapunov-type inequalities for nonlinear systems*, J. Math. Anal. Appl. **332** (2007), 497–511.
- [22] A. TIRYAKI, *Recent developments of Lyapunov-type inequalities*, Adv. Dyn. Syst. Appl. **5**, 2 (2010), 231–248.
- [23] M. ÜNAL, D. ÇAKMAK, A. TIRYAKI, *A discrete analogue of Lyapunov-type inequalities for nonlinear systems*, Comput. Math. Appl. **55** (2008), 2631–2642.
- [24] M. ÜNAL, D. ÇAKMAK, *Lyapunov-type inequalities for certain nonlinear systems on time scales*, Turkish J. Math. **32** (2008), 255–275.
- [25] X. WANG, *Stability criteria for linear periodic Hamiltonian systems*, J. Math. Anal. Appl. **367** (2010), 329–336.
- [26] A. WINTNER, *On the nonexistence of conjugate points*, Amer. J. Math. **73** (1951), 368–380.
- [27] X. YANG, Y. KIM, K. LO, *Lyapunov-type inequality for a class of odd-order differential equations*, J. Comput. Appl. Math. **234** (2010), 2962–2968.
- [28] X. YANG, Y. KIM, K. LO, *Lyapunov-type inequality for a class of quasilinear systems*, Math. Comput. Modelling **53** (2011), 1162–1166.
- [29] X. YANG, K. LO, *Lyapunov-type inequality for a class of even-order differential equations*, Appl. Math. Comput. **215** (2010), 3884–3890.