LANDAU AND GRÜSS TYPE INEQUALITIES FOR INNER PRODUCT TYPE INTEGRAL TRANSFORMERS IN NORM IDEALS

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Abstract. For a probability measure \( \mu \) and for square integrable fields \( (\mathcal{A}_t) \) and \( (\mathcal{B}_t) \) of commuting normal operators we prove Landau type inequality

\[
\left\| \int_{\Omega} \mathcal{A}_t X \mathcal{B}_t \, d\mu(t) - \int_{\Omega} \mathcal{A}_t \, d\mu(t) \int_{\Omega} \mathcal{B}_t \, d\mu(t) \right\| \leq \sqrt{\int_{\Omega} |\mathcal{A}_t|^2 \, d\mu(t) - \left( \int_{\Omega} \mathcal{A}_t \, d\mu(t) \right)^2} \cdot \left( \sqrt{\int_{\Omega} |\mathcal{B}_t|^2 \, d\mu(t) - \left( \int_{\Omega} \mathcal{B}_t \, d\mu(t) \right)^2} \right)
\]

for all \( X \in \mathcal{B}(\mathcal{H}) \) and for all unitarily invariant norms \( \| \cdot \| \).

For Schatten \( p \)-norms similar inequalities are given for arbitrary double square integrable fields. Also, for all bounded self-adjoint fields satisfying \( C \leq \mathcal{A}_t \leq D \) and \( E \leq \mathcal{B}_t \leq F \) for all \( t \in \Omega \) and some bounded self-adjoint operators \( C, D, E, F \), and for all \( X \in \mathcal{C} \| \cdot \| (\mathcal{H}) \) we prove Grüss type inequality

\[
\left\| \int_{\Omega} \mathcal{A}_t X \mathcal{B}_t \, d\mu(t) - \int_{\Omega} \mathcal{A}_t \, d\mu(t) \int_{\Omega} \mathcal{B}_t \, d\mu(t) \right\| \leq \frac{\|D - C\| \cdot \|F - E\|}{4} \cdot \|X\|.
\]

More general results for arbitrary bounded fields are also given.

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