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LANDAU AND GRÜSS TYPE INEQUALITIES FOR INNER PRODUCT TYPE INTEGRAL TRANSFORMERS IN NORM IDEALS

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Abstract. For a probability measure μ and for square integrable fields $(\mathscr{A}_t)$ and $(\mathscr{B}_t)$ $(t\in\Omega)$ of commuting normal operators we prove Landau type inequality

$$\left|\left|\left|\int_\Omega \mathscr{A}_t X \mathscr{B}_t d\mu(t) - \int_\Omega \mathscr{A}_t\, d\mu(t) X \int_\Omega \mathscr{B}_t\, d\mu(t)\right|\right|\right| \leqslant \left|\left|\left|\sqrt{\int_\Omega |\mathscr{A}_t|^2\, d\mu(t) - \left|\int_\Omega \mathscr{A}_t\, d\mu(t)\right|^2}\, X \sqrt{\int_\Omega |\mathscr{B}_t|^2\, d\mu(t) - \left|\int_\Omega \mathscr{B}_t\, d\mu(t)\right|^2}\right|\right|\right|$$

for all $X\in\mathfrak{B}(\mathscr{H})$ and for all unitarily invariant norms $|||\cdot|||$.

For Schatten p-norms similar inequalities are given for arbitrary double square integrable fields. Also, for all bounded self-adjoint fields satisfying $C\leqslant\mathscr{A}_t\leqslant D$ and $E\leqslant\mathscr{B}_t\leqslant F$ for all $t\in\Omega$ and some bounded self-adjoint operators C,D,E and F, and for all $X\in\mathfrak{C}_{|||\cdot|||}(\mathscr{H})$ we prove Grüss type inequality

$$\left|\left|\left|\int_\Omega \mathscr{A}_t X \mathscr{B}_t\, d\mu(t) - \int_\Omega \mathscr{A}_t\, d\mu(t) X \int_\Omega \mathscr{B}_t\, d\mu(t)\right|\right|\right| \leqslant \frac{\|D-C\|\cdot\|F-E\|}{4}\cdot|||X|||.$$

More general results for arbitrary bounded fields are also given.

Mathematics subject classification (2010): Primary 47A63; Secondary 46L05, 47B10, 47A30, 47B15.
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