Mathematical
Inequalities
& Applications
Volume 16, Number 1 (2013), 159–174
doi:10.7153/mia-16-12

WENDROFF TYPE INEQUALITIES ON TIME SCALES VIA PICARD OPERATORS

SZILÁRD ANDRÁS AND ALPÁR MÉSZÁROS

Abstract. Recently (see [11]) R.A.C. Ferreira, D.F.M. Torres proved some linear and nonlinear Wendroff type inequalities on time scales. Similar results were proved also by D.R. Anderson ([2] and [3]). It is well known (see [9]) that the Wendroff inequality is not the best possible upper estimate for the solutions of the corresponding integral inequality. The aim of our paper is to improve the known Wendroff type inequalities on time scales and to give a different proof for the existing inequalities. This improvement is motivated also by the work of A. Abdeldaim and M. Yakout (see [1] and [5]). The method we use is based on a variant of the abstract comparison Gronwall lemma (see [18], [15]) and on the theory of Picard operators ([16]).

Mathematics subject classification (2010): Primary 35A23, 26D10; Secondary 47H10, 45G10.
Keywords and phrases: Gronwall type inequality, Wendroff inequality, Picard operators, time scale.

REFERENCES

[1] A. ABDELDAIM AND M. YAKOUT, *On Wendroff's Inequality and Applications*, Int. Journal of Math. Analysis **4** (2010), 607–616.
[2] DOUGLAS R. ANDERSON, *Dynamic double integral inequalities in two independent variables on time scales*, Journal of Mathematical Inequalities **2**, 2 (2008), 163–184.
[3] DOUGLAS R. ANDERSON, *Nonlinear Dynamic Integral Inequalities in two Independent Variables on Time Scale Pairs*, Advances in Dynamical Systems and Applications **3**, 1 (2008), 1–13.
[4] SZ. ANDRÁS, *Ecuaţii integrale Fredholm-Volterra*, Ed. Didactică şi Pedagogică, Bucureţi, 2005.
[5] SZ. ANDRÁS, A. MÉSZÁROS, *Remarks on some Wendroff type inequalities*, Journal of Mathematical Inequalities **5**, 3 (2011), 401–411.
[6] E. F. BECKENBACH AND R. BELLMAN, *Inequalities*, Springer-Verlang, Berlin, 1961.
[7] M. BOHNER, A. PETERSON, *Dynamic equations on time scales. An introduction with applications*, Birkhäuser Boston, Inc., Boston, MA, 2001.
[8] M. BOHNER AND A. PETERSON, editors, *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, 2003.
[9] C. CRĂCIUN AND N. LUNGU, *Abstract and concrete Gronwall lemmas*, Fixed Point Theory **10**, 2 (2009), 221–228.
[10] S. S. DRAGOMIR, *On nonlinear integral inequalities of Gronwall type in two variables*, Journal of the Indonesian Mathematical Society **9**, 2 (2003), 77–87.
[11] R. A. C. FERREIRA AND D. F. M. TORRES, *Some linear and nonlinear integral inequalities on time scales in two independent variable*, Nonlinear Dynamics and Systems Theory **9**, 2 (2009), 161–169.
[12] S. HILGER, *Ein Makettenkalkul mit Anwendung auf Zentrumsmannigfaltigkeiten*, PhD thesis, Universität Würzburg, 1988.
[13] T. KULIK, C.C. TISDELL, *Volterra Integral Equations on Time Scales: Basic Qualitative and Quantitative Results with Applications to Initial Value Problems on Unbounded Domains*, International Journal of Difference Equations (IJDE) **3**, 1 (2008), 103–133.
[14] B. G. PACHPATTE, *Inequalities for Differential and Integral Equations*, Academic Press, New York and London, 1998.
[15] I. A. RUS, *Fixed points, upper and lower fixed points: abstract Gronwall lemmas*, Carpathian J. Math. **20**, 1 (2004), 125–134.

Paper MIA-16-12

[16] I. A. RUS, *Picard operators and applications*, Scienticae Mathematicae Japonocae **58**, 1 (2003), 1, 191–219.
[17] I. A. RUS, *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca, 2001.
[18] M. ŞERBAN AND C. CRĂCIUN, *A nonlinear integral equation via Picard operators*, Fixed Point Theory **12**, 1 (2011), 57–70.
[19] C. C. TISDELL AND A. ZAIDI, *Basic qualitative and quantitative results for solutions to nonlinear dynamic equations on time scales with an application to economic modelling*, Nonlinear Analysis: Theory, Methods & Applications **68**, 11, 3504–3524.