

## INTERPOLATIONS OF SCHWAB–BORCHARDT MEAN

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*Abstract.* For two means  $M, N$  satisfying  $M(x, y) \leq N(x, y)$  we apply the 'borchardtisation' process to obtain a new mean

$$SB_{M,N} = \frac{\sqrt{N^2 - M^2}}{\arccos(M/N)}.$$

We use some geometric ideas to prove inequalities between the three means. In particular some new inequalities for Seiffert means are established.

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