

ON COMPLETE MONOTONICITY OF SOME FUNCTIONS RELATED TO MEANS

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Abstract. We show the complete monotonicity of some functions related to the Stolarsky mean which was a problem of S.-X. Chen and F. Qi (2007) [15]. In the proof, the connection between operator monotone and completely monotonic functions is used.

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