ON THE DIRICHLET PROBLEM FOR THE GENERALIZED
n–LAPLACIAN: SINGULAR NONLINEARITY WITH THE EXPONENTIAL
AND MULTIPLE EXPONENTIAL CRITICAL GROWTH RANGE

ROBERT ČERNÝ

Abstract. Let $\Omega \subset \mathbb{R}^n$, $n \geqslant 2$, be a bounded domain containing the origin. Applying the Mountain Pass Theorem and a singular version of the generalized Moser-Trudinger inequality we prove the existence of a non-trivial weak solution to the problem

$$u \in W_0^1 L^\Phi(\Omega) \qquad \text{and} \qquad -\operatorname{div}\left(\Phi'(|\nabla u|)\frac{\nabla u}{|\nabla u|}\right) = \frac{f(x,u)}{|x|^a} \quad \text{in } \Omega\,,$$

where $a \in [0,n)$, Φ is a Young function such that the space $W_0^1 L^\Phi(\Omega)$ is embedded into exponential or multiple exponential Orlicz space and $f(x,t)$ has the corresponding critical growth.

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