MAJORIZATION UNDER CONSTRAINTS AND BOUNDS ON THE SECOND ZAGREB INDEX

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Abstract. In this paper we present a theoretical analysis in order to establish maximal and minimal vectors with respect to the majorization order of particular subsets of $\Re^n$. Afterwards we apply these issues to the calculation of bounds for a topological descriptor of a graph known as the second Zagreb index. Finally, we show how our bounds may improve the results obtained in the literature, providing some theoretical and numerical examples.

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