

NEW PERTURBATION BOUNDS FOR NONNEGATIVE AND POSITIVE POLAR FACTORS

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Abstract. The changes in the nonnegative and positive polar factors of generalized polar decomposition and polar decomposition are studied under the additive perturbation. Some new perturbation bounds are obtained. These bounds are different from previous ones in form and measure and, in some cases, may be smaller than the corresponding existing ones. Furthermore, the corresponding perturbation bounds for generalized nonnegative and positive polar factors of (M, N) weighted polar decomposition are also presented as the straightforward corollaries.

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