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STRICT VERIFICATION OF APPROXIMATE MIDCONVEXITY ON NON–CONVEX SETS

KRZYSZTOF MISZTAL AND JACEK TABOR

Abstract. Let V be a subset of an Abelian group G and let $\omega : V \times V \to [0,\infty]$ be given. We say that a function $f\colon V \to \mathbb{R}$ is $\omega(\cdot,\cdot)$*-midconvex* if

$$f(x) \leqslant \frac{f(x-\delta)+f(x+\delta)}{2} + \omega(x-\delta, x+\delta)$$

for $x \in V, \delta \in G$ such that $x-\delta, x+\delta \in V$. Our aim is to provide a computer assisted method to estimate

$$\sup\{f \in V \to \mathbb{R}\colon f \in \mathscr{B}(V;W), f \text{ is } \omega(\cdot,\cdot)\text{-midconvex}\},$$

where $\mathscr{B}(V;W)$ denotes the set of real-valued, bounded from above functions on V which are zero on W $(W \subset V)$. We present an algorithm which for given $\varepsilon > 0$ enables us, under reasonable assumptions, to find the above supremum with accuracy ε. We test our results for $V = \{0, \frac{1}{N}, \ldots, \frac{N-1}{N}, 1\}$ and $W = \{0,1\}$, where $N \in \mathbb{N}$ is fixed.

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Paper MIA-16-29

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