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NECESSARY AND SUFFICIENT CONDITIONS FOR SYMMETRIC HOMOGENEOUS POLYNOMIAL INEQUALITIES IN NONNEGATIVE REAL VARIABLES

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Abstract. Let $f_n(x,y,z)$ be a symmetric homogeneous polynomial of degree n. In this paper, we give the necessary and sufficient conditions to have $f_n(x,y,z) \geqslant 0$ for $n \leqslant 6$ and any nonnegative real numbers x,y,z. In addition, we extend some results to $n=7$ and $n=8$, and then apply the proposed method to prove several elegant symmetric homogeneous polynomial inequalities of degree n, $4 \leqslant n \leqslant 8$.

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