

## EMBEDDINGS PROPERTIES ON HERZ-TYPE BESOV AND TRIEBEL-LIZORKIN SPACES

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*Abstract.* We study the embeddings problems on Herz-type Besov-Triebel-Lizorkin spaces. In particular we will give a proof of the Sobolev-type embedding for these function spaces. All these results generalize the classical results on Besov and Triebel-Lizorkin spaces.

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