

INEQUALITIES INVOLVING LOGARITHMIC MEAN OF ARBITRARY ORDER

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Abstract. Inequalities involving logarithmic mean of arbitrary order are obtained. These results are derived from the Lazarević and Cusa-Huygens inequalities for hyperbolic functions. Some inequalities for the weighted sums of powers are also utilized. In particular cases obtained results simplify to known inequalities for the logarithmic mean of a low order.

Mathematics subject classification (2010): Primary: 26D15. Secondary: 26D05.

Keywords and phrases: Logarithmic mean of arbitrary order, power means, Lazarević and Cusa-Huygens inequalities, hyperbolic functions, inequalities.

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