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NORM INEQUALITIES FOR SOME ONE–SIDED OPERATORS

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Abstract. We show that the one-sided maximal operators associated with Borel measures are of strong type (p,p), $1 < p < \infty$, with constant p^*, and the related one-sided geometric maximal operators are of strong type (p,p), $0 < p < \infty$, with constant $e^{1/p}$. We also investigate norm inequalities for integral operators with three measures on the cone of nonnegative nonincreasing functions. Our results show that if we restrict the measures in the inequalities to some particular classes, then a simple characterization for these inequalities to hold can be obtained.

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