

AN APPROXIMATION PROPERTY OF POWER FUNCTIONS

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Abstract. We will solve the inhomogeneous linear first order differential equation of the form, $xy'(x) + \lambda y(x) = \sum_{m=0}^{\infty} a_m(x-c)^m$, and prove an approximation property of power functions. More precisely, we prove the local Hyers-Ulam stability of linear first order differential equation, $xy'(x) + \lambda y(x) = 0$, in a special class of analytic functions.

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REFERENCES

- [1] C. ALSINA AND R. GER, *On some inequalities and stability results related to the exponential function*, J. Inequal. Appl. **2** (1998), 373–380.
- [2] S. CZERWIK, *Functional Equations and Inequalities in Several Variables*, World Sci. Publ., Singapore, 2002.
- [3] P. GÄVRUȚA, S.-M. JUNG AND Y. LI, *Hyers-Ulam stability for second-order linear differential equations with boundary conditions*, Electron. J. Differential Equations **2011**, 80 (2011), 1–5.
- [4] D. H. HYERS, G. ISAC AND TH. M. RASSIAS, *Stability of Functional Equations in Several Variables*, Birkhäuser, Boston, 1998.
- [5] S.-M. JUNG, *Legendre's differential equation and its Hyers-Ulam stability*, Abst. Appl. Anal. **2007** (2007), Article ID 56419, 14 pages, doi: 10.1155/2007/56419.
- [6] S.-M. JUNG, *Approximation of analytic functions by Hermite functions*, Bull. Sci. math. (2007), doi: 10.1016/j.bulsci.2007.11.001.
- [7] S.-M. JUNG, *An approximation property of exponential functions*, Acta Math. Hungar. **124**, 1–2 (2009), 155–163.
- [8] S.-M. JUNG, *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*, Springer, New York, 2011.
- [9] W. KOSMALA, *A Friendly Introduction to Analysis – Single and Multivariable* (2nd edn), Pearson Prentice Hall, London, 2004.
- [10] T. MIURA, S.-M. JUNG AND S.-E. TAKAHASI, *Hyers-Ulam-Rassias stability of the Banach space valued linear differential equations $y' = \lambda y$* , J. Korean Math. Soc. **41** (2004), 995–1005.
- [11] M. OBŁOZA, *Hyers stability of the linear differential equation*, Rocznik Nauk.-Dydakt. Prace Mat. **13** (1993), 259–270.
- [12] M. OBŁOZA, *Connections between Hyers and Lyapunov stability of the ordinary differential equations*, Rocznik Nauk.-Dydakt. Prace Mat. **14** (1997), 141–146.
- [13] M. H. PROTTER AND C. B. MORREY, *A First Course in Real Analysis* (2nd edn), Springer, New York, 1991.
- [14] S.-E. TAKAHASI, T. MIURA AND S. MIYAJIMA, *On the Hyers-Ulam stability of the Banach space-valued differential equation $y' = \lambda y$* , Bull. Korean Math. Soc. **39** (2002), 309–315.
- [15] W. R. WADE, *An Introduction to Analysis* (2nd edn), Prentice Hall, Upper Saddle River, NJ, 2000.