

SET VALUED F -VARIATIONAL INEQUALITIES AND SET VALUED VECTOR F -COMPLEMENTARITY PROBLEMS

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Abstract. In this work, we study a new type of set-valued vector F -variational inequalities and a new kind of set valued vector F -complementarity problem in Hausdorff topological vector spaces. We establish the equivalence between the set valued vector F -variational inequalities and set valued vector F -complementarity problems under certain conditions. By considering the existence of solutions for the vector F -variational inequalities and using the continuous selection theorem, we obtain some new existence theorems of solutions for the set valued vector F -variational inequalities and set valued vector F -complementarity problems, respectively.

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