

SOME NEW SCALES OF REFINED HARDY TYPE INEQUALITIES VIA FUNCTIONS RELATED TO SUPERQUADRACITY

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Abstract. For the Hardy type inequalities the “breaking point” (= the point where the inequality reverses) is $p = 1$. Recently, J. Oguntose and L. E. Persson proved a refined Hardy type inequality with a breaking point at $p = 2$. In this paper we prove a new scale of refined Hardy type inequality which can have a breaking point at any $p \geq 2$. The technique is to first make some further investigations for superquadratic and superterzatic functions of independent interest, among which, a new Jensen type inequality is proved.

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