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A CHARACTERIZATION OF THE STABILITY OF A SYSTEM OF THE BANACH SPACE VALUED DIFFERENTIAL EQUATIONS

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Abstract. We will consider the Banach space valued differential equation $\mathfrak{y}'(t) = A\mathfrak{y}(t)$, where A is an $n \times n$ complex matrix. We give a necessary and sufficient condition in order that the equation have the Hyers-Ulam stability. As a Corollary, we prove that the Banach space valued linear differential equation with constant coefficients $y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \cdots + a_1 y'(t) + a_0 y(t) = 0$ has the Hyers-Ulam stability if and only if $\mathrm{Re}\,\lambda \neq 0$ for all the solutions λ of the equation $z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0 = 0$.

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