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COMMUTATORS FOR MULTIPLIERS ON BESOV DUNKL SPACES

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Abstract. In this paper, we first study the boundedness properties of the Dunkl multiplier of the interval $[a,b]$ associated with the reflection group $\mathbb{Z}_2$. Next, we prove that the commutator $[T,T_\mu]$ is bounded on the Besov Dunkl spaces $BD_p^{\sigma,q}$, if T is a bounded linear operator on $BD_p^{\sigma_j,q_j}$ $(j=0,1$ and $0<\sigma_1<\sigma<\sigma_0)$ and T_μ is a dyadic admissible multiplier. These results are obtained for the multi-dimensional Dunkl transform associated to the reflection group $\mathbb{Z}_2^d$.

Mathematics subject classification (2010): Primary 33C52; secondary 42B10, 43A32, 33C80, 22E30.
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