

A NOTE ON THE TRIANGLE INEQUALITY FOR THE C^* -VALUED NORM ON A HILBERT C^* -MODULE

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Abstract. The C^* -valued norm is defined on a Hilbert C^* -module by its standard inner product. In this short note, we will prove that $|x+y| \leq |x| + |y|$ holds for all $x, y \in E$ which is a Hilbert \mathcal{A} -module if and only if $\mathcal{J} = \overline{\langle E, E \rangle}$, the closed two-sided ideal in \mathcal{A} , is a commutative C^* -algebra.

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