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SCHUR POWER CONVEXITY OF THE DARÓCZY MEANS

ZHEN-HANG YANG

Abstract. In this paper, the Schur convexity is generalized to Schur f-convexity, which contains the Schur geometrical convexity, harmonic convexity and so on. When $f:\mathbb{R}_+\to\mathbb{R}$ is defined by $f(x)=(x^m-1)/m$ if $m\neq 0$ and $f(x)=\ln x$ if $m=0$, the necessary and sufficient conditions for f-convexity (is called Schur m-power convexity) of Daróczy means are given, which improve, generalize and unify Shi et al.'s results.

Mathematics subject classification (2010): 26D15, 26D99, 26B25.
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