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OPTIMAL POLYNOMIAL BOUNDS FOR THE EXPONENTIAL FUNCTION

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Abstract. We find polynomial lower and upper bounds of e^x on some respective intervals. To be specific, for each natural number n, we construct polynomials $p_n(x)$ and $q_n(x)$ of degree n so that $p_n(x) \leqslant p_{n+1}(x) \leqslant e^x$ and $e^x \leqslant q_{n+1}(x) \leqslant q_n(x)$ on some intervals, respectively. These polynomials are optimal in the sense that if $p(x)$ (or $q(x)$) is a polynomial of degree n with $p_{n-1}(x) \leqslant p(x) \leqslant e^x$ (or $e^x \leqslant q(x) \leqslant q_{n-1}(x)$) then $p(x) \leqslant p_n(x)$ (or $q_n(x) \leqslant q(x)$). The fact that $1/p_n(-x)$ works as an upper bound of e^x on a switched interval is interesting. We also provide the size comparison between two upper bounds $q_n(x)$ and $1/p_n(-x)$.

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