MATRIX YOUNG NUMERICAL RADIUS INEQUALITIES

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Abstract. In the present paper, we show that if $A \in M_n(\mathbb{C})$ is a non scalar strictly positive matrix such that $1 \in \sigma(A)$, and $p > q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$, then there exists $X \in M_n(\mathbb{C})$ such that $\omega(AXA) > \omega(\frac{1}{p}A^pX + \frac{1}{q}XA^q)$. Moreover, several numerical radius inequalities are presented for Hilbert space operators. In particular, we prove that if $p \geqslant q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$, then

$$\omega^r(A^*XB) \leqslant \left\| \frac{1}{p}(A^*|X^*|A)^{\frac{rp}{2}} + \frac{1}{q}(B^*|X|B)^{\frac{rq}{2}} \right\|, \text{ for all } A,B,X \in \mathcal{B}(\mathbf{H}) \text{ and } r \geqslant \frac{2}{q}.$$

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