Mathematical
Inequalities
& Applications
Volume 16, Number 3 (2013), 823–829
doi:10.7153/mia-16-62

OPTIMAL HÖLDER MEAN INEQUALITY FOR THE COMPLETE ELLIPTIC INTEGRALS

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Abstract. In this paper, we prove that $H_p(\mathscr{K}(r),\mathscr{K}(r')) \geqslant \mathscr{K}(\sqrt{2}/2)$ and $H_q(\mathscr{K}(r),\mathscr{K}(r')) \leqslant \mathscr{K}(\sqrt{2}/2)$ for all $r \in (0,1)$ if and only if $p \geqslant 1-4[\mathscr{K}(\sqrt{2}/2)]^4/\pi^2 = -3.789\cdots$ and $q \leqslant (\log 2)/[\log(\pi/2) - \log \mathscr{K}(\sqrt{2}/2)] = -4.1805\cdots$, where $H_p(x,y)$ denotes the Hölder mean of order p of two positive numbers x and y, $r' = \sqrt{1-r^2}$, and $\mathscr{K}(r)$ denotes the complete elliptic integral of the first kind, respectively.

Mathematics subject classification (2010): Primary 26E60.
Keywords and phrases: Complete elliptic integrals, Hölder mean, inequality.

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Paper MIA-16-62