OPTIMAL HÖLDER MEAN INEQUALITY
FOR THE COMPLETE ELLIPTIC INTEGRALS

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Abstract. In this paper, we prove that
\[
H_p(\mathcal{K}(r), \mathcal{K}(r')) \geq \mathcal{K}(\sqrt{2}/2)
\]
and
\[
H_q(\mathcal{K}(r), \mathcal{K}(r')) \leq \mathcal{K}(\sqrt{2}/2)
\]
for all \( r \in (0, 1) \) if and only if
\[
p \geq 1 - 4\left[ \mathcal{K}(\sqrt{2}/2) \right]^{4/\pi^2} = -3.789 \cdots
\]
and
\[
q \leq \log(2)/\left[ \log(\pi/2) - \log(\mathcal{K}(\sqrt{2}/2)) \right] = -4.1805 \cdots
\]
where \( H_p(x, y) \) denotes the Hölder mean of order \( p \) of two positive numbers \( x \) and \( y \), \( r' = \sqrt{1 - r^2} \), and \( \mathcal{K}(r) \) denotes the complete elliptic integral of the first kind, respectively.

Keywords and phrases: Complete elliptic integrals, Hölder mean, inequality.

REFERENCES