IMPROVED INEQUALITIES FOR TRIGONOMETRIC
FUNCTIONS VIA DIRICHLET AND ZETA FUNCTIONS

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Abstract. The article discusses several improvements of well known inequalities for trigonometric functions. We utilize the monotonicity of the Riemann zeta function, as well as the Dirichlet eta, beta and lambda functions, to shorten the proofs of known inequalities for trigonometric functions, and to obtain new ones.

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