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SOLUTIONS OF THE MATRIX INEQUALITIES IN THE MINUS PARTIAL ORDERING AND LÖWNER PARTIAL ORDERING

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Abstract. Two matrices A and B of the same size are said to satisfy the minus partial ordering, denoted by $B \leqslant^{-} A$, iff the rank subtractivity equality $\operatorname{rank}(A-B) = \operatorname{rank}(A) - \operatorname{rank}(B)$ holds; two complex Hermitian matrices A and B of the same size are said to satisfy the Löwner partial ordering, denoted by $B \leqslant^{\mathrm{L}} A$, iff the difference $A-B$ is nonnegative definite. In this note, we establish general solution of the inequality $BXB^{*} \leqslant^{-} A$ induced from the minus partial ordering, and general solution of the inequality $BXB^{*} \leqslant^{\mathrm{L}} A$ induced from the Löwner partial ordering, respectively, where $(\cdot)^{*}$ denotes the conjugate transpose of a complex matrix. As consequences, we give closed-form expressions for the shorted matrices of A relative to the range of B in the minus and Löwner partial orderings, respectively, and show that these two types of shorted matrices in fact are the same.

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