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CARLEMAN ESTIMATES AND UNIQUE CONTINUATION PROPERTY FOR ELLIPTIC OPERATORS IN BANACH SPACES

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Abstract. The unique continuation theorems for elliptic differential-operator equations in Banach-valued L_p-space are investigated. The operator-valued multiplier theorems and the Carleman estimates for the equations are employed to obtain these results. In applications the unique continuation theorems for anisotropic elliptic differential equations and finite or infinite systems of elliptic equations are studied.

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