

NONLINEAR GRONWALL–BELLMAN TYPE INTEGRAL INEQUALITIES WITH MAXIMA

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Abstract. Integral inequalities with maxima of the unknown function are useful in the study of control theory. Known results were given for Gronwall-Bellman type integral inequalities with the maxima in the form of linear dependence on the unknown function with a single delay term. In this paper we consider a general form of nonlinear integral inequalities with the maxima and more than one delay terms. Requiring neither monotonicity nor separability of given functions, we apply monotonicization to estimate the unknown function. Our result can be used to weaken conditions for some known results. We apply our result to prove boundedness of solutions for a differential equation with the maxima and an integral equation with maxima separately.

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