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# OSCILLATION OF $p(x)$–LAPLACIAN ELLIPTIC INEQUALITIES WITH MIXED VARIABLE EXPONENTS

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*Abstract.* Oscillation criteria are established for $p(x)$-Laplacian elliptic inequalities with mixed variable nonlinearities of the form

$$u\big[\nabla\cdot(A(x)|\nabla u|^{p(x)-2}\nabla u)+\langle b(x),|\nabla u|^{p(x)-2}\nabla u\rangle-h(x,u)+g(x,u)\big]\leqslant 0,\quad x\in\Omega,$$

where $\beta(x)>p(x)>\gamma(x)>1$, $\Omega$ is an exterior domain in $\mathbb{R}^N$, and

$$\begin{aligned}h(x,u)&=\ln|u|\,|\nabla u|^{p(x)-2}\,(A(x)\nabla u)\cdot\nabla p(x),\\ g(x,u)&=c(x)|u|^{p(x)-2}u+c_1(x)|u|^{\beta(x)-2}u+c_2(x)|u|^{\gamma(x)-2}u+f(x).\end{aligned}$$

The function $h(x,u)$ recently introduced in [N. Yoshida, Nonlinear Anal. 74 (2011) 2563–2575] allows employing the Riccati transformation technique commonly used in the oscillation theory of ordinary differential equations.

It should be noted that the results obtained are new for one dimensional case as well. Examples are given to illustrate the results.

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