

SCHUR CONVEXITY OF DUAL FORM OF THE COMPLETE SYMMETRIC FUNCTION

KONGSHENG ZHANG AND HUANNAN SHI

Abstract. In this paper, the Schur-convexity, the Schur-geometric-convexity and the Schur-harmonic-convexity of dual form of the complete symmetric function are investigated. As consequences, some new inequalities are established via majorization theory.

Mathematics subject classification (2010): 26B25, 05E05.

Keywords and phrases: complete symmetric function, dual form, Schur-convexity, Schur-geometric-convexity, Schur-harmonic-convexity.

REFERENCES

- [1] Y.-M. CHU AND Y.-P. LV, *The Schur harmonic convexity of the Hamy symmetric function and its applications*, J. Inequal. Appl., 2009, Art. ID. 838529, 10 pages.
- [2] Y.-M. CHU AND T.-C. SUN, *The Schur harmonic convexity for a class of symmetric functions*, Acta Math. Sci. Ser. B Engl. Ed., **30** (2010), 1501–1506.
- [3] Y.-M. CHU, G.-D. WANG AND X.-H. ZHANG, *The Schur multiplicative and harmonic convexities of the complete symmetric function*, Math. Nachr., **284** (2011), 653–663.
- [4] Y.-M. CHU AND W.-F. XIA, *Necessary and sufficient conditions for the Schur harmonic convexity of the generalized Muirhead mean*, Proc. A. Razmadze Math. Inst., **152** (2010), 19–27.
- [5] Y.-M. CHU, W.-F. XIA AND X.-H. ZHANG, *The Schur concavity, Schur multiplicative and harmonic convexities of the second dual form of the Hamy symmetric function with applications*, J. Multivariate Anal., **105** (2012), 412–421.
- [6] Y.-M. CHU, W.-F. XIA AND T.-H. ZHAO, *Schur convexity for a class of symmetric functions*, Sci. China Math., **53** 2010, 465–474.
- [7] Y.-M. CHU AND X.-M. ZHANG, *Necessary and sufficient conditions such that extended mean values are Schur-convex or Schur-concave*, J. Math. Kyoto Univ., **48** (2008), 229–238.
- [8] Y.-M. CHU, X.-M. ZHANG AND G.-D. WANG, *The Schur geometrical convexity of the extended mean values*, J. Conv. Anal., **15** (2008), 707–718.
- [9] K.-Z. GUAN, *Schur-convexity of the complete symmetric function*, Math. Inequal. Appl., **9** (2006), 567–576.
- [10] K.-Z. GUAN, *Some properties of a class of symmetric functions*, J. Math. Anal. Appl., **33** (2007), 670–680.
- [11] K.-Z. GUAN, *The Hamy symmetric function and its generalization*, Math. Inequal. Appl., **9** (2006), 797–805.
- [12] K.-Z. GUAN AND J.-H. SHEN, *Schur-convexity for a class of symmetric function and its applications*, Math. Inequal. Appl., **9** (2006), 199–210.
- [13] W.-D. JIANG, *Some properties of dual form of the Hamy's symmetric function*, J. Math. Inequal., **1** (2007), 117–125.
- [14] B.-Y. LONG AND Y.-M. CHU, *The Schur convexity and inequalities for a class of symmetric functions*, Acta Math. Sci. Ser. A Chin. Ed., **32** (2012), 80–89.
- [15] A. W. MARSHALL AND I. OLKIN, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.
- [16] J.-X. MENG, Y.-M. CHU AND X.-M. TANG, *The Schur-harmonic-convexity of dual form of the Hamy symmetric function*, Matematicki Vesnik, **62** (2010), 37–46.

- [17] I. SCHUR, *Über eine Klasse von Mittelbildungen mit Anwendungen auf die Determinantentheorie*, Sitzungsber Berlin Math Ges, **22** (1923), 9–20.
- [18] J. N. WHITELEY, *Some inequalities concerning symmetric forms*, *Mathematika*, **5** (1958), 47–49.
- [19] W.-F. XIA AND Y.-M. CHU, *On Schur-convexity of some symmetric functions*, *J. Inequal. Appl.*, 2010, Art. ID. 543250, 12 pages.
- [20] W.-F. XIA AND Y.-M. CHU, *Schur convexity and Schur multiplicative convexity for a class of symmetric functions with applications*, *Ukrainian Math.*, **61** (2009), 1541–1555.
- [21] W.-F. XIA AND Y.-M. CHU, *Schur-convexity for a class of symmetric functions and its applications*, *J. Inequal. Appl.*, 2009, Art. ID. 493759, 15 pages.
- [22] W.-F. XIA AND Y.-M. CHU, *Schur convexity with respect to a class of symmetric functions and their applications*, *Bull. Math. Anal. Appl.*, **3** (2011), 84–96.
- [23] W.-F. XIA AND Y.-M. CHU, *The Schur convexity of Gini mean values in the sense of harmonic mean*, *Acta Math. Sci. Ser. B, Engl. Ed.*, **31** (2011), 1103–1112.
- [24] W.-F. XIA, Y.-M. CHU AND G.-D. WANG, *Necessary and sufficient conditions for the Schur harmonic convexity or concavity of the extended mean values*, *Rev. Un. Mat. Argentina*, **52** (2011), 121–132.
- [25] W.-F. XIA, G.-D. WANG AND Y.-M. CHU, *Schur convexity and inequalities for a class of symmetric functions*, *Int. J. Pure Appl. Math.*, **58** (2010), 435–452.
- [26] X.-M. ZHANG, *Geometrically Convex Functions*, Anhui University Press, Hefei, 2004 (in Chinese).
- [27] X.-M. ZHANG, *S-Geometric convexity of a function involving Maclaurin's elementary symmetric mean*, *J. Inequal. Pure. Appl. Math.*, Vol. 8, 2007, Article 51, 6 pages.