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AN EXPERIMENTAL CONJECTURE INVOLVING CLOSED–FORM EVALUATION OF SERIES ASSOCIATED WITH THE ZETA FUNCTIONS

JUNESANG CHOI AND HARI M. SRIVASTAVA

Abstract. The subject of closed-form summation of series involving the Zeta functions has been remarkably widely investigated. Recently, in the course of his trying to give a closed-form expression for the Dirichlet beta function $\beta(2n)$ $(n \in \mathbb{N})$, Lima [16] posed a very interesting experimental conjecture for a closed-form evaluation of a certain class of series involving the Riemann Zeta function $\zeta(s)$. Here, in the present sequel to Lima's work, we aim at verifying correctness of Lima's conjecture and presenting several general analogues of Lima's conjecture. Our demonstration and derivations are based mainly upon a known formula for series associated with the Zeta functions. Relevant connections of some specialized results of the main identities presented here with those obtained in earlier works are also pointed out.

Mathematics subject classification (2010): Primary 11M35, 11M36; Secondary 11M06, 33B15.
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