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CVAR–BASED FORMULATION AND APPROXIMATION METHOD FOR A CLASS OF STOCHASTIC VARIATIONAL INEQUALITY PROBLEMS

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Abstract. In this paper, we consider CVaR-based formulation and approximation method proposed by Chen and Lin [5] for a class of stochastic variational inequality problems (for short, SVIP). Different from the work mentioned above, we regard the regularized gap function for SVIP as a loss function for SVIPs and obtain a restrained deterministic minimization reformulation for SVIPs. We show that the reformulation is a convex program for a wider class of SVIPs than that in [5]. Furthermore, by using the smoothing techniques and Monte Carlo method, we get an approximation problem of the minimization reformulation and consider the convergence of optimal solutions and stationary points of the approximation problems. Finally we apply our proposed model to solve the migration equilibrium problem under uncertainty.

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Paper MIA-16-77

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