

## ON log MAJORIZATIONS FOR POSITIVE SEMIDEFINITE MATRICES

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*Abstract.* In this paper, we show that every log majorization for positive semidefinite matrices can be expressed in countable sets of log majorizations and all of them are equivalent to one another.

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