AN ADDITIVE FUNCTIONAL INEQUALITY
IN MATRIX NORMED SPACES

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Abstract. Using the fixed point method and the direct method, we prove the Hyers-Ulam stability of an additive functional inequality in matrix normed spaces.

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