ON THE HYERS—ULAM STABILITY OF SEXTIC FUNCTIONAL EQUATIONS IN β–HOMOGENEOUS PROBABILISTIC MODULAR SPACES

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Abstract. In this paper, we present a fixed point method to prove the generalized Hyers–Ulam stability of the systems of additive-quadratic-cubic functional equations with constant coefficients in β-homogeneous probabilistic modular spaces.

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