

STABILITY OF CAUCHY ADDITIVE FUNCTIONAL EQUATION IN FUZZY BANACH SPACES

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Abstract. In this article, we prove the generalized Hyers–Ulam stability of the following Cauchy additive functional equation

$$f\left(\frac{x-y}{n}+z\right)+f\left(\frac{y-z}{n}+x\right)+f\left(\frac{z-x}{n}+y\right)=f(x+y+z)$$

in fuzzy Banach spaces for any fixed nonzero integer n .

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REFERENCES

- [1] T. AOKI, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan. **2** (1950), 64–66.
- [2] T. BAG AND S. K. SAMANTA, *Finite dimensional fuzzy normed linear spaces*, J. Fuzzy Math. **11** (2003), 687–705.
- [3] T. BAG AND S. K. SAMANTA, *Fuzzy bounded linear operators*, J. Fuzzy Math. **151** (2005), 513–547.
- [4] L. CĂDARIU AND V. RADU, *Fixed points and the stability of Jensen’s functional equation*, J. Inequal. Pure Appl. Math. **4** (2003), no. 1, Art. 4, 7 pages.
- [5] S. C. CHENG AND J. M. MORDESON, *Fuzzy linear operators and fuzzy normed linear spaces*, Bull. Calcutta Math. Soc. **86** (1994), 429–436.
- [6] C. FELBIN, *Finite dimensional fuzzy normed linear spaces*, Fuzzy Sets and Systems **48** (1992), 239–248.
- [7] P. GĂVRUTA, *A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.
- [8] O. HADŽIĆ, E. PAP AND V. RADU, *Generalized contraction mapping principles in probabilistic metric spaces*, Acta Math. Hungar. **101** (2003), no. 1–2, 131–148.
- [9] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Natl Acad. Sci. **27** (1941), 222–224.
- [10] G. ISAC AND TH. M. RASSIAS, *Stability of ψ –additive mappings*, Applications to nonlinear analysis, Internat. J. Math. Sci. **19** (1996), 219–228.
- [11] A. K. KATSARAS, *Fuzzy topological vector spaces II*, Fuzzy Sets and Systems **12** (1984), 143–154.
- [12] I. KRAMOSIL AND J. MICHALEK, *Fuzzy metric and statistical metric spaces*, Kybernetika **11** (1975), 336–344.
- [13] S. V. KRISHNA AND K. K. M. SARMA, *Separation of fuzzy normed linear spaces*, Fuzzy Sets and Systems **63** (1994), 207–217.
- [14] B. MARGOLIS AND J. B. DIAZ, *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc. **126** (1968), 305–309.
- [15] D. MIHET AND V. RADU, *On the stability of the additive Cauchy functional equation in random normed spaces*, J. Math. Anal. Appl. **343** (2008), 567–572.
- [16] A. K. MIRMOSTAFAEI, M. MIRZAVAZIRI AND M. S. MOSLEHIAN, *Fuzzy stability of the Jensen functional equation*, Fuzzy Sets Systems **159** (2008), 730–738.

- [17] A. K. MIRMOSTAFEE AND M. S. MOSLEHIAN, *Fuzzy versions of Hyers-Ulam-Rassias theorem*, Fuzzy Sets and Syst. **159** (2008), 720–729.
- [18] M. S. MOSLEHIAN AND A. NAJATI, *An application of a fixed point theorem to a functional inequality*, Fixed Point Theory **10** (2009), 141–149.
- [19] C. PARK, *Fixed points and Hyers–Ulam–Rassias stability of Cauch–Jensen functional equations in Banach algebras*, Fixed point Theory and Appl. **2007** (2007), Art. ID 50175, 15 pages.
- [20] V. RADU, *The fixed point alternative and the stability of functional equations*, Fixed point Theory **4** (2003), 91–96.
- [21] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Am. Math. Soc. **72** (1978), 297–300.
- [22] S. M. ULAM, *A collection of Mathematical Problems*, Interscience Publ. New York, 1960.
- [23] J. Z. XIAO AND X. H. ZHU, *Fuzzy normed spaces of operators and its completeness*, Fuzzy Sets and Systems **133** (2003), 389–399.