

STRONG CONVERGENCE THEOREMS OF STRONGLY RELATIVELY NONEXPANSIVE SEMI-GROUP IN BANACH SPACES

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Abstract. In this paper, we prove some strong convergence approximation theorems of strongly relatively nonexpansive semi-group in the framework of Banach spaces. Using the concept of duality theorems, we obtain analogue results for strongly generalized nonexpansive semi-group and for semi-group of firmly generalized nonexpansive type. The results presented in this paper improve and extend some corresponding results announced by some authors.

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