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STRONG CONVERGENCE THEOREMS FOR BREGMAN QUASI–ASYMPTOTICALLY NONEXPANSIVE MAPPINGS AND EQUILIBRIUM PROBLEM IN REFLEXIVE BANACH SPACES

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Abstract. The purpose of this article is to propose an iteration algorithm for Bergman quasi-asymptotically nonexpansive mapping to have the strong convergence under a limit condition only in the framework of reflexive Banach spaces. As applications, we apply our results to a system of equilibrium problems. The results presented in the paper improve and extend the corresponding results of Reich and Sabach [Nonlinear Anal. 73 (2010), 122–135], Suantai et al. [Comput. Math. Appl. 64 (2012), 489–499], Nilsrakoo and Saejung [Appl. Math. Comput. 217:14 (2011), 6577–6586], Qin et al. [Applied Math Letters, 22 (2009), 1051–1055], Wang et al. [J. Comput. Appl. Math., 235 (2011), 2364–2371], Su et al. [Nonlinear Anal. 73 (2010), 390–3906], Nartinez-Yanes et al. [Nonlinear Anal., 64 (2006), 2400–2411] and others.

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