TRIPLED BEST PROXIMITY POINT THEOREM IN METRIC SPACES

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Abstract. The purpose of this article is to first introduce the notion of tripled best proximity point and cyclic contraction pair. We also establish the existence and convergence theorems of tripled best proximity points in metric spaces. Moreover, we apply our results to setting of uniformly convex Banach space. Finally, we obtain some results on the existence and convergence of tripled fixed point in metric spaces and give illustrative examples of our theorems.

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