GENERALIZATION OF JENSEN'S INEQUALITY BY LIDSTONE'S POLYNOMIAL AND RELATED RESULTS

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Abstract. In this paper we consider $(2n)$-convex functions and completely convex functions. Using Lidstone's interpolating polynomials and conditions on Green's functions we present results for Jensen's inequality and converses of Jensen's inequality for signed measure. By using the obtained inequalities, we produce new exponentially convex functions. Finally, we give several examples of the families of functions for which the obtained results can be applied.

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