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# OPPENHEIM'S PROBLEM AND RELATED INEQUALITIES FOR DUNKL KERNELS

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*Abstract.* In this paper, we establish some inequalities related to Oppenheim's problem for Dunkl kernels. In order to prove our main results, we present new inequalities involving modified Bessel functions of the first kind. Refinements of inequalities for modified Bessel functions are also given.

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*Keywords and phrases*: Oppenheim's problem, Bessel functions, modified Bessel functions, Dunkl kernels.

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