Mathematical
Inequalities
& Applications
Volume 17, Number 1 (2014), 75–81
doi:10.7153/mia-17-04

INTEGRABILITY AND BOUNDEDNESS OF EXTREMAL FUNCTIONS OF A HARDY–SOBOLEV TYPE INEQUALITY

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Abstract. In this paper, we study the properties of positive solutions of an integral equation in R^n

$$u(x) = \int_{R^n} \frac{u^\gamma(y)dy}{|x-y|^{n-\alpha}|y|^{-\sigma}}, \quad x \in R^n.$$

Such a nonlinear singular equation is related to the study of the best constant of the Hardy-Sobolev type inequality. According to the Newton potential theory, this integral equation is helpful to understand the Henon type partial differential equation when $\alpha = 2$. We use the weighted Hardy-Littlewood-Sobolev inequality to obtain the optimal integrability interval of positive integrable solutions. Namely, if $u \in L^{\frac{2n}{n-\alpha-\sigma}}(R^n)$, then $u \in L^t(R^n)$ for all $t > \frac{n}{n-\alpha-\sigma}$. Based on this result, we prove that those integrable solutions must be bounded.

Mathematics subject classification (2010): 26D15, 45E10, 45G05.
Keywords and phrases: Hardy-Sobolev type inequality, extremal functions, integrability interval, weighted Hardy-Littlewood-Sobolev inequality.

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