INTEGRABILITY AND BOUNDEDNESS OF EXTREMAL FUNCTIONS OF A HARDY–SOBOLEV TYPE INEQUALITY

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Abstract. In this paper, we study the properties of positive solutions of an integral equation in $\mathbb{R}^n$

$$u(x) = \int_{\mathbb{R}^n} \frac{u(y)dy}{|x-y|^{n-\alpha} |y|^{-\sigma}}, \quad x \in \mathbb{R}^n.$$  

Such a nonlinear singular equation is related to the study of the best constant of the Hardy-Sobolev type inequality. According to the Newton potential theory, this integral equation is helpful to understand the Henon type partial differential equation when $\alpha = 2$. We use the weighted Hardy-Littlewood-Sobolev inequality to obtain the optimal integrability interval of positive integrable solutions. Namely, if $u \in L^\frac{2n}{n-\alpha-\sigma}(\mathbb{R}^n)$, then $u \in L^t(\mathbb{R}^n)$ for all $t \geq \frac{n}{n-\alpha-\sigma}$. Based on this result, we prove that those integrable solutions must be bounded.


Keywords and phrases: Hardy-Sobolev type inequality, extremal functions, integrability interval, weighted Hardy-Littlewood-Sobolev inequality.

REFERENCES


